# Queueing Theory Application at Ticket Windows in An Indian Railway Station 



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#### Abstract

\section*{Abstract}

In everyday life we encounter situations where we have to face a long queue. In any service system, long queue is a big problem. India railway is the life line of public transport. So, in this paper analysis of queue at the ticket windows of an Indian railway station was carried out. In India, number of passengers keeps growing. So, length of waiting queue on ticket window also keeps increasing. Also due to delay and rescheduling of trains, number of waiting passengers increases. Here we used queueing theory, for the estimation of waiting time on the basis of real time arrival, service and departure data for solution of problems related to long queue at ticket windows in railway station. To achieve this, first we obtained primary data on the basis of observation. Then we used appropriate queueing model to analyze the system and calculate the waiting time as result. In this paper, results show that increasing the number of server lead to decrease in average waiting time of passengers in queue and decrease in average waiting time of passengers in system. Therefore, by applying the queueing theory, the waiting time of passengers can be improved. As per result, increasing the number of server (i.e. number of ticket windows) can solve long queue problem at railway station. Also, number of ticket window opened at a time can be varied according to number of arrivals in this system for efficient utilization of servers. The result can used by the management in railway station to increase the efficiency of operations at their ticket windows.


Keywords: Arrival Rate, Service Rate, Traffic Intensity, Performance Measures, Waiting Times.

## Introduction

The railways in India are the largest rail web in Asia and the world's 3rd largest rail networks. The Indian Railways route length network has $115,000 \mathrm{~km}$ of track length. It runs 12,617 trains to carry over 23 million passengers daily - equivalent to moving the entire population of Australia - connecting more than 7,172 stations. Railways runs more than 7,421 freight trains carrying 3 million tones (MT) of freight every day. India's railway network is recognized as one of the largest railway systems in the world under single management. In 1845, along with Sir Jamsetjee Jejeebhoy, Hon. Jaganath Shunkerseth (known as Nana Shankarsheth) formed the Indian Railway Association. Lord Dalhousie is known was the father of Indian Railways. The railway network is also ideal for longdistance travel and movement of bulk commodities, apart from being an energy efficient and economic mode of conveyance and transport. Railway transportation is a major form of land-based transportation. Rail transport is means of transferring of passengers and goods on wheeled vehicles running on rails, also known as tracks. Railway transportation has a much potential because of many advantages like safety, affordability, dependability and capability of transferring of peoples, goods and services. In India railway transportation has been existence since 1853. The first commercial train journey in India between Bombay and Thane on 16 April 1853 in a 14-carriage long train drawn. A railway station consists of at least one-track side platform and station building. Various operations are carried out inside the station building. The ticket operation is one of the most important operations carried out in a railway station. In big station ticketing officers attend to passenger through a window known as 'ticket window'. At tickets windows, tickets purchased by a passenger enable access to service desired while payment received from passengers. In some railway station, ticket windows are not sufficient, this results in large queues. In some cases, server is idle because passenger is not seen at ticket window for long periods. In some railway station, ticketing agent attending first the passengers they know while presence of other passengers that arrived
earlier and case of negative passenger (like collusion or jockeying) increase a waiting line at ticket windows of railway station. The fast forward world of technology everyone is running behind the time. The main motivation of technology is to produce time and costefficient products. In railway department online ticket booking or e- ticketing was introduced. By using this facility, railway passenger to book ticket on internet via a government website like IRCTC.CO.IN. The printout of tickets may be used for validation. After that using of online ticket booking and e-ticket, waiting time of passengers may be reduced in railway ticket window. Inability of management in station to determine the waiting time of passengers at their ticket window results in large queues at ticket windows. In this paper, we study the waiting line of passengers at ticket window of Gwalior railway station. Here we apply queueing model to improve the waiting time of passengers in ticket operations.

## Review of Literature

Xu , Gao and Ou (2007) [9] researched on "Service performance analysis and improvement for a ticket queue with balking customers", in this paper they were focusing on queuing systems managed by ticket technology and showing that different ticket queues have different balking probabilities when customer patience is low and system traffic is high. They proposed improvement in ticket queue technology, viewing it as a new area for researchers to explore. S. Vijay Prasad and V. H. Badshah [7] has suggested an Alternate queuing system for tatkal railway reservation system to decrease Customer's waiting time.

Ronald R. Gilliam [5] described an application that can help the organization design passenger screening facilities to reduce the waiting time of customers. Moving a step further, Ghosal,Chaturvedi, Taywade and Jaisankar (2015)[1] in their work on "Android application for ticket booking and ticket checking in suburban railways" proposed the use of online ticketing applications, mobile phones and other devices as a remedy to stress and tension endured by passengers standing in long queues to book tickets. After that we study Umanah and Udoh [8] research paper that is "queueing theory application at ticket window railway station (a study of Lagos terminus IDDO, Lagos state, Nigeria). In this paper author obtained a primary data by the observation and applying queueing model then calculated the result. in this paper researcher trying to do decrease waiting time of passengers at ticket windows in Lagos terminus.

## Methodology

Here we study the waiting time of passengers at the ticket window in Gwalior railway station. In our study first we obtain the primary data on the basis of observation of the queuing system at the ticket windows in the Gwalior railway station Terminus. The study population is made up of all passengers that come to the railway station to purchase tickets and all servers provide the ticketing to passengers. The sample for the study is conveniently drawn from all passengers and servers
involved in the ticketing operation from day shift (8 hours), Monday to sunday, for a weak.

## Model Specification

The models of Queuing theory are used to represent various types of queuing systems in real life. Basically, queuing models are describing the behavior of queuing system. Here the appropriate queuing model for this study is:

M/M/c/ $\infty /$ FCFS
This model describes a queuing system with Poisson probability distributed arrival from an infinite calling population, exponentially distributed service time, multiple but identical servers in parallel, infinite system capacity and a first-come first-served queue discipline. We shall make the following general assumptions for queueing system:

1. Arrival of passengers follow a Poisson probability distribution.
2. Passenger arrival are independent and the arrival rate do not change overtime.
3. Service times are exponentially distributed and the mean service rate was constant for each server.
4. A single waiting line are formed and each arrival waited to be served according to the queue length.
5. The system had multiple but identical servers in parallel.
6. No passenger left the queue without being served.
7. Infinite capacity of the system.
8. First-come first-served queue discipline

Let $\lambda=$ arrival rate
$\mu=$ service rate
$\mathrm{c}=$ number of servers
and $\rho=$ Traffic intensity ( $=\frac{\lambda}{C \mu}$ ),

## Probability that the system is idle (Po)

$$
P_{0}=\left[\sum_{\mathrm{n}=0}^{\mathrm{c}-1} \frac{(\mathrm{c} \rho)^{\mathrm{n}}}{\mathrm{n}!}+\frac{1}{\mathrm{c}!}\left(\frac{\mathrm{c} \rho^{\mathrm{c}}}{1-\rho}\right)\right]^{-1}
$$

The average number of passengers in queue $\left(\mathrm{L}_{\mathrm{q}}\right)$

$$
\mathrm{Lq}=\left[\frac{1}{(\mathrm{c}-1)!}\left(\frac{\lambda}{\mu}\right)^{\mathrm{c}}\left(\frac{\lambda \mu}{(\mathrm{c} \mu-\lambda)^{2}}\right)\right] \mathrm{P}_{0}
$$

The average number of passengers in the system ( $\mathrm{L}_{\mathrm{S}}$ )

$$
L_{S}=L q+\frac{\lambda}{\mu}
$$

The average waiting time for a passenger in queue $\left(W_{\mathrm{q}}\right)$

$$
\mathrm{Wq}=\frac{\mathrm{L}_{\mathrm{q}}}{\lambda}
$$

The average waiting time for a passenger in the system ( $\mathrm{W}_{\mathrm{S}}$ )

$$
\mathrm{W}_{\mathrm{S}}=\mathrm{Wq}+\frac{1}{\mu}
$$

The numbers of servers (c), as well as the values of the mean arrival rate ( $\lambda$ ), mean service rate $(\mu)$ and the traffic intensity ( $\rho$ ), are necessary in determining the performance measures for the model.

# Shrinkhla Ek Shodhparak Vaicharik Patrika 

## Results and Finding

Presentation of analyzed data
Table 1
Presentation and Summary Analysis of Primary Data for Passenger's Behavior at Ticket Window of A Railway

| Inputs | Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of arrived <br> Passengers | 1050 | 1435 | 1355 | 1345 | 1325 | 1119 | 930 |
| Number of served <br> Passengers | 1044 | 1427 | 1348 | 1338 | 1319 | 1111 | 925 |
| Total time of <br> observation (hours) | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| c | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| $\lambda$ | 131.25 | 179.37 | 169.37 | 168.12 | 165.62 | 139.87 | 116.25 |
| $\mu$ | 130.5 | 178.37 | 168.5 | 167.25 | 164.87 | 138.87 | 115.62 |
| $\rho$ | 0.5028 | 0.5028 | 0.5025 | 0.5026 | 0.5022 | 0.5036 | 0.5027 |
| $\mathrm{P}_{0}$ | 0.3308 | 0.3308 | 0.3311 | 0.3310 | 0.3313 | 0.3301 | 0.3309 |
| $\mathrm{~L}_{\mathrm{q}}$ | 0.3404 | 0.3401 | 0.3397 | 0.3344 | 0.3389 | 0.3421 | 0.3400 |
| $\mathrm{~L}_{\mathrm{S}}$ | 1.3461 | 1.3457 | 1.3448 | 1.3396 | 1.3434 | 1.3493 | 1.3454 |
| $\mathrm{~W}_{\mathrm{q}}$ | 0.0025 | 0.0018 | 0.0020 | 0.0019 | 0.0020 | 0.0024 | 0.0029 |
| $\mathrm{~W}_{\mathrm{S}}$ | 0.0101 | 0.0074 | 0.0079 | 0.0078 | 0.0080 | 0.0096 | 0.0115 |
| $\mathrm{To}^{2}$ |  |  |  |  |  |  |  |

To have a view of queueing situation, the entire 7 days is considered and analyzed using the same method. Here we use the queuing model for our study as referred to as $\mathrm{M} / \mathrm{M} / \mathrm{c}$ queueing model, Where $\mathrm{c}=$ the number of server (ticket window) $=2$

Table 2
Analysis of the Data Showing Passenger Behavior at Three Ticket Windows of Railway Station

| Inputs | Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of arrived <br> Passengers | 1840 | 2400 | 2352 | 2340 | 2280 | 2112 | 1792 |
| Number of served <br> Passengers | 1827 | 2392 | 2348 | 2332 | 2272 | 2108 | 1788 |
| Total time of <br> observation (hours) | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| c | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| $\lambda$ | 230 | 300 | 294 | 292.5 | 285 | 264 | 224 |
| $\mu$ | 228.375 | 299 | 293.5 | 291.5 | 284 | 263.5 | 223.5 |
| $\rho$ | 0.3357 | 0.3344 | 0.3339 | 0.3344 | 0.3345 | 0.3339 | 0.3340 |
| $\mathrm{P}_{0}$ | 0.3612 | 0.3624 | 0.3629 | 0.3624 | 0.3623 | 0.3629 | 0.3628 |
| $\mathrm{~L}_{\mathrm{q}}$ | 0.0467 | 0.0460 | 0.0457 | 0.0460 | 0.0461 | 0.0457 | 0.0458 |
| $\mathrm{~L}_{\mathrm{S}}$ | 1.0538 | 1.0493 | 1.0474 | 1.0495 | 1.0496 | 1.0476 | 1.0481 |
| $\mathrm{~W}_{\mathrm{q}}$ | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0002 |
| $\mathrm{~W}_{\mathrm{S}}$ | 0.0045 | 0.0034 | 0.0035 | 0.0035 | 0.0036 | 0.0038 | 0.0046 |

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Table 3

| Analysis of The Data Showing Passenger Behavior At Four Ticket Windows Of Railway Station |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Inputs | Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 |
| Number of arrived <br> Passengers | 2816 | 2788 | 2512 | 327002 | 3298 | 3138 | 2708 |
| Number of served <br> Passengers | 2808 | 2783 | 2500 | 3265 | 3289 | 3128 | 2696 |
| Total time of <br> Observation(hours) | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| c | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| $\lambda$ | 352 | 348.5 | 314 | 409 | 412.25 | 392.25 | 338.5 |
| $\mu$ | 351 | 347.87 | 312.5 | 408.12 | 411.12 | 391 | 337 |
| $\rho$ | 0.2507 | 0.2504 | 0.2512 | 0.2505 | 0.2506 | 0.2507 | 0.2511 |
| $\mathrm{P}_{0}$ | 0.3663 | 0.3667 | 0.3655 | 0.3666 | 0.3664 | 0.3663 | 0.3665 |
| $\mathrm{~L}_{\mathrm{q}}$ | 0.0068 | 0.0068 | 0.0070 | 0.0068 | 0.0068 | 0.0069 | 0.0069 |
| $\mathrm{~L}_{\mathrm{S}}$ | 1.0096 | 1.0086 | 1.0118 | 1.0089 | 1.0095 | 1.0100 | 1.0113 |
| $\mathrm{~W}_{\mathrm{q}}$ | 0.00001 | 0.00001 | 0.00002 | 0.00001 | 0.00001 | 0.00001 | 0.00001 |
| $\mathrm{~W}_{\mathrm{S}}$ | 0.0028 | 0.0028 | 0.0032 | 0.0024 | 0.0024 | 0.0025 | 0.0029 |

Table 4
Study of Waiting Time of Customers in Queues with Increase in Number Of Servers

| INPUTS | $\mathbf{c}=\mathbf{2}$ | $\mathbf{c}=\mathbf{3}$ | $\mathbf{c}=\mathbf{4}$ |
| :---: | :---: | :---: | :---: |
| $\rho$ | 0.5027 | 0.3344 | 0.2507 |
| $\mathrm{~W}_{\mathrm{q}}$ (hours) | 0.0022 | 0.0001 | 0.00001 |
| $\mathrm{~W}_{\mathrm{S}}$ (hours) | 0.0089 | 0.0038 | 0.0027 |

As servers increased from two to three, the average waiting time of a passenger in queue decreased from 0.0022 hours ( 7.97 seconds) to 0.0001 hours ( 0.36 seconds) which represents a total of $95.45 \%$ decrease. Also, the average waiting time of a passenger in the system decreased from 0.0089 hours ( 32.83 seconds) to 0.0038 hours ( 13.83 seconds), representing 57.30\% decrease inwaiting. Furthermore, when the servers increased from three to four, Wq decreased from 0.0001 hours ( 0.36 seconds) to 0.00001 hour ( 0.036 seconds) representing a total of $90 \%$ decrease in waiting, while Ws decreased from 0.0038 hours ( 13.83 seconds) to 0.0027 hours ( 9.72 seconds) representing a $28.95 \%$ decrease in waiting. This is very important, as it shows that alterations can be applied to improve the queuing system and reduce waiting time of passengers.
Conclusion
This paper represents the importance of applying queuing theory in solving waiting line problems at ticket windows in railway stations. In this study result shows that the waiting times of passengers can be calculated and improved upon if necessary. The results also show that servers can be assessed and judgments on their service rates are less difficult to make when queuing theory is applied.

Based on our study, we concluded that proper analysis of the queuing system at the ticket windows by the management of railway stations can lead to a reduction in waiting lines at ticket windows in Indian railway station. It can improve efficiency of the entire ticketing operation and many passengers can be satisfied by the services of management of railway station. The Indian Railway Corporation (IRC) can improve access to tickets by selling them in public places (other than their railway stations), through certified agentand on the internet, IRC approved online platform. This will encourage better facilities and reduce waiting line at ticket window in an Indian railway station.

## End notes

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